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COMPARISON OF STEADY-STATE SVC MODELS IN LOAD FLOW CALCULATIONS

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Abstract—This paper compares in a load flow calculation three existing steady-state models of static var compensator (SVC), i.e. the generator-fixed susceptance model, the total susceptance model and the firing angle model. The comparison is made in terms of the voltage at the SVC regulated bus, equivalent SVC susceptance at the fundamental frequency and the load flow convergence rate when SVC is operating both within and on the limits. The latter two models give inaccurate results of the SVC susceptance due to the assumption of constant voltage when the SVC is operating within the limits. This may underestimate the SVC regulating capability. Two modified models are proposed to improve the SVC regulated voltage according to its steady-state characteristic. The simulation results of the two modified models show the improved accuracy of the SVC susceptance while retaining acceptable load flow convergence rate.

I. INTRODUCTION

Flexible ac transmission systems (FACTS) utilize high power semiconductor devices to control the reactive power flow and thus the active power flow of the transmission system so that the ac power can be transmitted through a long distance efficiently [1]. Nowadays, FACTS devices are also used in high-voltage distribution systems. There are two main categories of FACTS devices, i.e. series-connected devices and shunt-connected devices. The series-connected FACTS device, such as thyristor controlled series compensator (TCSC), is to connect in series a controllable reactance in a power network so that the electrical distance between the generators and the load centers is controlled. The shunt-connected FACTS device, such as static var compensator (SVC) and static synchronous condenser (STATCON), is to regulate bus voltages or to compensate local reactive power consumption by injecting reactive power to electrical networks. There are also combined series and shunt FACTS devices, such as unified power flow controllers. In order to analyze the performance of modern power systems integrated with various FACTS devices, the understanding of the steady-state and dynamic interaction between the FACTS controllers and the electrical systems are necessary.

For steady-state analysis of electrical systems, the standard tool is a load flow calculation using the Newton-Raphson method. For electrical systems integrated with FACTS controllers, an accurate representation of FACTS devices is crucial to determine the appropriate location and preliminary rating of the devices as well as to study their effects on the system power flows and voltages under normal, abnormal and

contingency conditions. In addition, the steady-state models of FACTS devices together with the load flow results provide the initial conditions for other power system analyses, such as harmonic analysis and stability issues due to large and small disturbances. Therefore, there are a number of steady-state models of FACTS devices proposed so far [2]–[4]. In terms of SVC, there are mainly three well-accepted steady-state models available, i.e. the generator-fixed susceptance model [2][3], the total susceptance model and the firing angle model [5][6].

This paper focuses on the steady-state modelling of SVC in load flow calculations. First of all, the steady-state characteristic of a SVC is discussed. Secondly, the three existing steady-state models for SVC are briefly discussed. The shortcomings of these models are also pointed out. Thirdly, a simple model based on the characteristic of the SVC is mentioned as well as its limitations. Then, two modified models based on the total susceptance model and firing angle model are presented. Finally, a simulation based on a 5-bus system is carried out to compare the results from the six SVC models, when the SVC is operating both within and on the limits. The comparison is made in terms of the voltage at the SVC regulated bus, the equivalent SVC impedance at the fundamental frequency and the convergence rate of the load flow algorithm.

II. STEADY-STATE CHARACTERISTIC OF A SVC

A simple SVC that works both in the capacitive and inductive range can be obtained by a fixed-capacitor (FC) in parallel with a thyristor controlled reactor (TCR). The FC-TCR is the most commonly used SVC device in practice. The SVC models discussed in this paper are based on the FC-TCR. However, all the steady-state SVC models in the load flow calculation to be discussed in the next section are applicable to other types of SVC, such as the thyristor switched capacitor (TSC)-TCR. The diagram of a FC-TCR is shown in Fig. 1.

The V-I diagram of the steady-state operation of a SVC is shown in Fig. 2, where E and X_s are the system equivalent Thevenin voltage and reactance, respectively. In practice, a SVC uses droop control of the voltage at the regulated bus. The droop or the slope shown in Fig. 2 is exaggerated for the sake of the clear illustration. Usually the slope is less than 5%. The droop control means that the voltage at the regulated bus is controlled within a certain interval $[V_{\min}, V_{\max}]$, instead of a constant voltage value V_{ref} . The relaxation of the constant

voltage control improves the performance of a SVC in a number of ways:

- For a given SVC susceptance range, the current rating of the capacitor can be reduced
- If the system's equivalent Thevenin voltage decreases from E_1 to E_2 due to, e.g. the increased system loading, a droop-controlled SVC requires a much smaller change in susceptance than a constant voltage controlled SVC, in which the susceptance already reaches its capacitive limit as shown in Fig. 2.
- If the system's equivalent Thevenin impedance X_s decreases further, as the dotted line shown in Fig. 2, the constant voltage controlled SVC is no longer able to provide any active support, and in this case behaves as a fixed capacitor. Under such condition, the SVC cannot help to improve the dynamic response of the system subjected to any transient event followed, e.g. a voltage dip due to a system fault. Whereas the droop-controlled SVC, which still remains within the regulated range, can actively participate in the improvement of the system dynamic behavior.

Considering all these aspects, an accurate representation of the SVC droop control during steady-state analysis is important, especially when the SVC is operating close to the limits. In addition, an accurate SVC susceptance or the corresponding firing angle is necessary for other power system analysis such as initializing SVC during a harmonic analysis. Therefore, how to accurately implement a realistic SVC model in a load flow algorithm is of great importance.

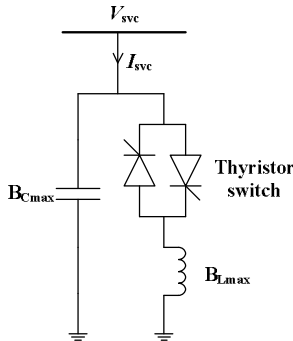


Fig. 1. Diagram of a SVC device.

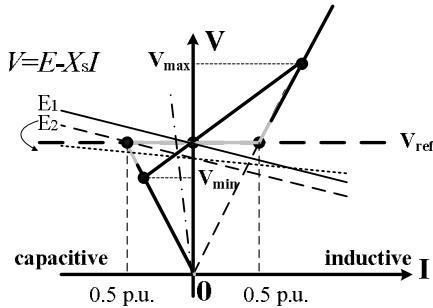


Fig. 2. Steady-state characteristic of a SVC.

III. SVC MODELS IN LOAD FLOW CALCULATIONS

There are mainly three existing SVC models in load flow calculations, e.g. the generator-fixed susceptance model, the total susceptance model and the firing angle model. This section first of all briefly discusses the three existing models. The inaccuracy caused by the total susceptance model and firing angle model is pointed out. A characteristic model based on the steady-state characteristic of SVC is then presented. In the end, two combined models are developed to improve the accuracy of the total susceptance model and the firing angle model.

A. The generator-fixed susceptance model

When SVC is operating within the limits, the relation between its injected reactive power and the regulated bus voltage follows the droop characteristic shown in Fig. 1. When the SVC is operating on the limits, it behaves like a fixed susceptance and its injected reactive power is proportional to the square of the regulated bus voltage. Therefore, the SVC cannot be simply modeled as a PV bus with fixed reactive power limits. The modeling of the droop control of a SVC is essential especially in the case of weak systems.

The generator model represents the slope by connecting the SVC to an auxiliary bus separated from the high voltage bus by a reactance equal to the per unit slope. The generator model of a SVC with and without a step-down transformer is shown in Fig. 3. The generator model can be directly used in a conventional load flow program. However, the generator model is valid only when the SVC is operating within the regulated limits. The SVC model has to be changed to a fixed capacitor or inductor model depending on the operating limit. The generator model needs 2 or 3 nodes (depending on without or with a step-down transformer) in a load flow program; whereas the fixed impedance model only needs 1 node. When the SVC is changing from the generator model to the fixed susceptance model, the load flow input data matrices need to be modified and the corresponding Jacobian matrix is re-dimensioned and reordered. In other words, a new load flow calculation is needed. The necessity to continuously check whether or not the SVC is changing between the generator model and the fixed-susceptance

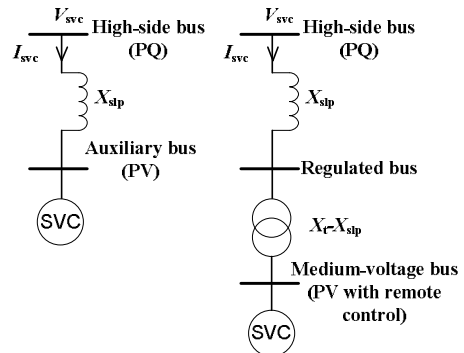


Fig. 3. SVC generator model: without (left) and with (right) step-down transformer.

model also makes the method inefficient and troublesome.

B. The total susceptance model and firing angle model

The total susceptance model developed by [5] incorporates the SVC model in an advanced load flow program. It represents the SVC model as an adjustable susceptance connected to the high-voltage bus with susceptance limits.

The total susceptance model assumes a fixed voltage at the SVC connection busbar when operating within the limits. Therefore, it is similar to a PV bus. However, instead of eliminating one order from the Jacobian matrix of the reactive power mismatch equation (due to the PV bus), another one representing the relation between the SVC injected reactive power Q_{SVC} and its equivalent susceptance B_{SVC} is used at the corresponding row of the Jacobian matrix [5]:

$$\Delta Q_{SVC}^k = (\partial Q_{SVC} / \partial B_{SVC})^k \Delta B_{SVC}^k \quad (1)$$

where k is the iteration number and

$$(\partial Q_{SVC} / \partial B_{SVC})^k = (V_{SVC}^k)^2 = Q_{SVC}^k / B_{SVC}^k. \quad (2)$$

In this way, the SVC susceptance B_{SVC} instead of the voltage becomes the state variable, and thus the SVC bus is called PVB node. The column in the Jacobian matrix corresponding to the partial derivative with respect to the voltage of the SVC bus should be set to zero except for the one, which is replaced by (2), at the row of the reactive power mismatch equation of the SVC bus. In addition, the calculated Q_{SVC}^k should also be subtracted from the corresponding power mismatch term. When the SVC reaches the limits, the PVB node is changed to a PQ node. In other words, the SVC becomes a fixed susceptance and the Jacobian needn't to be modified as when operating within the limits. The merit of the total susceptance model is that it only needs one node for the SVC in the load flow algorithm when operating both within and on the limits.

The first problem of the total susceptance model is that it assumes the SVC voltage to be constant when operating within the limits. This may cause an error of the final SVC susceptance value due to the ignorance of the SVC slope. The second one is that the assumption alters the moment when the SVC reaches its operating limits. Both problems will be shown in the next section.

The firing angle model can be seen as an extension of the total susceptance model, but with the firing angle as the state variable directly instead of the SVC susceptance. This is particularly useful when the firing angle is needed for the initialization of harmonic analysis. The total susceptance model, however, needs another iterative process to obtain the firing angle from the susceptance value. The Jacobian matrix corresponding to the partial derivative of the reactive power with respect to the firing angle is modified as [5]:

$$\Delta Q_{SVC}^k = (\partial Q_{SVC} / \partial \alpha_{SVC})^k \Delta \alpha_{SVC}^k \quad (3)$$

where α_{SVC} is the firing angle of the SVC and

$$(\partial Q_{SVC} / \partial \alpha_{SVC})^k = 2(V_{SVC}^k)^2 (\cos(2\alpha_{SVC}^k) - 1) / (\pi X_{Lmax}) \quad (4)$$

and X_{Lmax} is the reactance of the inductor.

The equivalent susceptance can be calculated by:

$$B_{SVC}^k = 1/X_{Cmax} - [2(\pi - \alpha_{SVC}^k) + \sin(2\alpha_{SVC}^k)] / (\pi X_{Lmax}) \quad (5)$$

where X_{Cmax} is the reactance of the capacitor.

In the case of a SVC connected through a transformer, the total susceptance model or the firing angle model can still be applied as the total impedance is the transformer impedance plus the equivalent SVC impedance [7].

C. The characteristic model

The characteristic model proposed is based on the linear relation of the SVC steady-state characteristic shown in Fig. 2. It also treats the SVC as an adjustable susceptance. Instead of modifying the Jacobian matrix, it modifies the admittance matrix with the updated SVC susceptance. The change of the SVC impedance at k^{th} iteration is calculated by:

$$\Delta B_{SVC}^k = (V_{SVC}^k - V_{ref}) / (s V_{SVC}^k) - B_{SVC}^k. \quad (6)$$

where s is the slope of the SVC characteristic. The SVC impedance at the next iteration is updated by:

$$B_{SVC}^{k+1} = B_{SVC}^k + \Delta B_{SVC}^k. \quad (7)$$

When the SVC reaches its limits, the susceptance is fixed at its maximum capacitive or inductive limit. The algorithm requires only 1 node when operating both within and on the limits and considers the droop control of the SVC characteristic. Besides, the algorithm is simple and straightforward. However, the problem with the characteristic model is that the convergence rate is relatively poor and not stable when the system condition is changed.

D. The combined models

The first combined model is based on the total susceptance model. However, instead of assuming a constant voltage when operating within the limits, it updates the voltage according to the droop characteristic of SVC given the updated susceptance. The voltage at the SVC bus is updated by:

$$V_{SVC}^k = V_{ref} / (1 - s B_{SVC}^k). \quad (8)$$

The combined model retains the merits of the total susceptance model, i.e. 1 node needed when operating both within and on the limits as well as a stable convergence rate. In addition, it provides a more accurate SVC susceptance value than the total susceptance model by updating the voltage at the SVC bus. The second combined model is to combine the firing angle model with the characteristic model. The approach is similar to the first combined model, i.e. to

update the SVC voltage by using (8) when the SVC is operating within the limits.

IV. COMPARISON OF SVC MODELS

A. Simulation case

The foregoing six SVC models are tested on a 5-bus network as shown in Fig. 4. The load flow calculation converges in 5 iterations with the maximum power mismatch of 10^{-12} . The obtained voltage at bus 4 is 0.920 p.u.. In order to regulate the voltage at bus 4 to 1 p.u., a SVC is connected to the bus. The maximum susceptances of the SVC in inductive and capacitive range are 3.472 p.u. and 0.935 p.u., respectively. The slope of the SVC is 1%. The SVC is able to regulate the voltage within [0.991, 1.036] p.u..

B. Within the operating limits

After the SVC is connected to bus 4, the results of the voltage at the bus with the six models are shown in Fig. 5. The equivalent SVC susceptances are shown in Fig. 6. The firing angles from the firing angle model and the corresponding combined model are shown in Fig. 7. The generator-fixed susceptance model, the total susceptance model, the firing angle model and the characteristic model are referred to as the G model, B model, F model and C model in the figures, respectively. The combined B and C model and the combined F and C model are referred to as the BC model and the FC model, respectively. The final voltage at bus 4, the equivalent SVC susceptance and the iteration number of the six models are summarized in Table I.

As the SVC is operating within the limits (in the capacitive range in this case), the generator model provides an accurate result, which can be taken as a reference to the other models. As both the total susceptance model and the firing angle model assume a constant voltage value when the SVC is operating within the limits, a final susceptance value of 0.414 p.u. is obtained by the two models instead of 0.414 p.u. by the generator model. Therefore, the two models do not accurately reflect the actual operating characteristic of the SVC. The characteristic model gives the same susceptance value as the generator model. The modified total susceptance model and firing angle model, i.e. the combined BC model and FC model, both give the same results as the generator model. This is due to the reason that the combined models modify the SVC voltage according to the SVC droop characteristic at each iteration instead of keeping the voltage constant.

As also shown in Table I, the generator model converges in 5 iterations. The total susceptance model and firing angle model converge in 5 and 6 iterations, respectively. The characteristic model converges in 21 iterations. The low convergence rate is due to the reason that the change of the susceptance is calculated from a linear relation between the susceptance and the voltage so that the quadratic convergence rate of a standard Newton-Raphson algorithm is reduced. The combined BC and FC model converge in 11 and 10 iterations, respectively. The convergence rate of the combined models is higher than that of the characteristic model. This is because

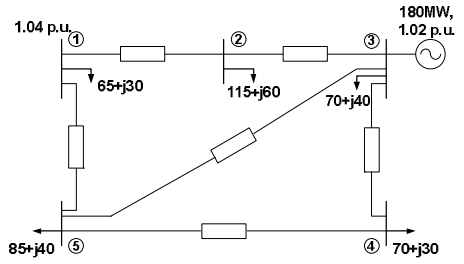


Fig. 4. 5-bus test system.

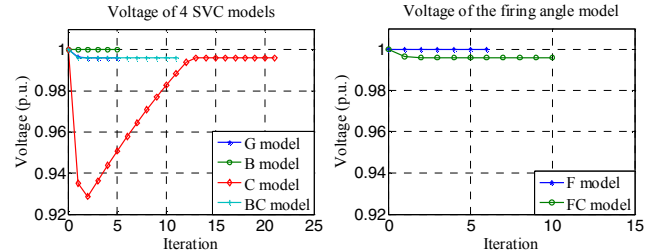


Fig. 5. Voltage at bus 4 from the six SVC models.

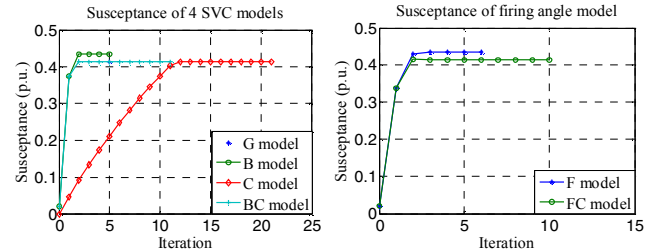


Fig. 6. Equivalent SVC susceptance from the six SVC models.

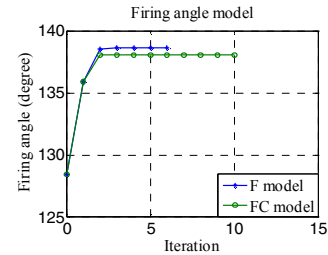


Fig. 7. Firing angle from the firing angle model and the combined model.

the change of the susceptance in the combined models is calculated from the Newton-Raphson algorithm, which keeps its quadratic convergence rate. The increased iteration number as compared to the generator model is due to the update of the SVC voltage, which is based on the SVC operating characteristic. The convergence rate of the six models will be further demonstrated in the next subsection.

TABLE I
VOLTAGE AT BUS 4, EQUIVALENT SVC SUSCEPTANCE AND ITERATION
NUMBER FROM THE SIX SVC MODELS

	G model	B model	F model	C model	BC model	FC model
V_4	0.996	1	1	0.996	0.996	0.996
B_{SVC} (a_{SVC})	0.414	0.435	0.435 (138.63°)	0.414	0.414	0.414 (138.02°)
Iteration	5	5	6	21	11	10

C. On the operating limits

In order to spot the edge of the SVC regulating limits, a simple continuation load flow is performed on the network with the load at bus 4 increasing at the step of 1% from 1 to 2 times the base value shown in Fig. 4. The voltage at bus 4 and the equivalent susceptance from the six models are shown in Fig. 8. As shown in the figure, the total susceptance model and firing angle model reach the SVC capacitive limit when the load at bus 4 is increased to $(126+j54)$ MVA while at the same time the voltage starts decreasing from 1 p.u. However, the generator model, the characteristic model and the two combined models all reach the SVC capacitive limit when the load at bus 4 is increased further to $(129.5+j55.5)$ MVA. The susceptance model and firing angle model give different results as compared to the other models due to the constant voltage assumption when operating within the limits. As the difference of the SVC voltage increases, the difference of the susceptance increases. The corresponding convergence rates of the six models when the load increases are shown in Fig. 9. It is worth pointing out that although the generator-fixed susceptance model seems to have the best convergence rate during the whole range of the load scale, it needs new load flow calculations when the SVC is changing between the generator model and the fixed susceptance model. However, the other models, no matter under which conditions, require only one load flow calculation. As shown in Fig. 9, the firing angle model has similar convergence rate as the total susceptance model. This also holds for the corresponding combined models. The combined BC (FC) model has slower convergence rate than the total susceptance (firing angle) model, especially when SVC is operating within the limits. The convergence rate of the characteristic model is exponentially decreasing when the SVC is approaching its capacitive limit and becomes too large to be acceptable when the SVC reaches its limit. However, the convergence rate of the combined models is acceptable.

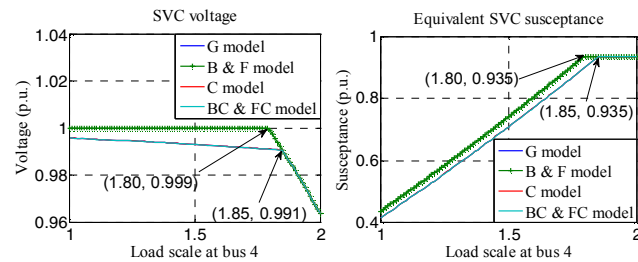


Fig. 8. SVC voltage and equivalent susceptance from the six SVC models with load scale at bus 4 of [1:0.01:2].

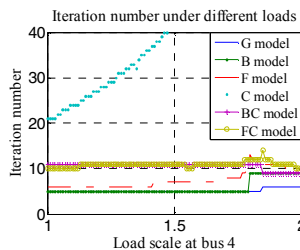


Fig. 9. Load flow convergence rate of the six SVC models with load scale at bus 4 of [1:0.01:2].

V. CONCLUSIONS

Six different existing models of SVC in load flow calculations have been presented. Both conditions, when SVC is operating within and on the limits, have been examined. The generator model is only valid when the SVC is operating within the regulating limits and has to change to the fixed-susceptance model when operating on the limits. This alters the total bus number of the whole network and new load flows need to be carried out if using the conventional load flow program. The total susceptance model and the firing angle model use the susceptance and the firing angle as the state variable, respectively, and can obtain the susceptance and/or firing angle within one load flow calculation. However, both models assume a constant voltage when the SVC is operating within the limits, and thus do not represent the realistic SVC operating characteristic. Consequently, the two models do not provide the actual equivalent susceptance value at the fundamental frequency. The characteristic model based on the steady-state droop control of the SVC is also discussed but fails to have an acceptable convergence rate. Two new models, the combined BC and FC model, provide an accurate value of the equivalent SVC susceptance; while the convergence rate of the two combined models is also acceptable.

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